Welcome to Class

-All cell phone in the holder

Warm-up

1. Is (x + 1) a factor of $5x^3 + x^2 - 5x - 1$?

2. If $x = -1$ is a solution factor the polynomial $5x^3 + 9x^2 + 3x - 1$.

$$\begin{array}{r|rrrr} -1 & 5 & 9 & 3 & -1 \\ & \downarrow & -5 & -4 & 1 \\ \hline & 5 & 4 & -1 & \boxed{0} \end{array}$$

$5x^2 + 4x - 1$

$(5x - 1)(x + 1)$

$1x$

$5x$

$\cancel{(x+1)(5x+1)(x-1)}$

$(5x-1)(x+1)^2$

$$\begin{array}{r} x = -1 \\ +1 \quad +1 \\ \hline x+1 = 0 \end{array}$$

The Rational Zero Theorem:

WHY IT IS IMPORTANT: **Narrows the search for rational zeros to a finite list.**
-It gives a place to start to find the answer.

- If $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ has integer coefficients $(a_n \neq 0)$ and $\frac{p}{q}$ is a rational zero (in lowest terms) of p, then **p is the factors of the constant term** a_0 **and q is the factors of the leading coefficient** a_n.

EX 1: Find the possible rational roots of $f(x) = x^3 + 6x^2 + 10x + 3$.

3 1

What is p? 1, 3 What is q? 1

List of **POSSIBLE** roots: $\frac{1}{1}, \frac{3}{1} \rightarrow \boxed{\pm 1, \pm 3}$

EX 2: Find the possible rational roots of $f(x) = 3x^3 + 5x^2 + 7x + 2$.

2 3

What is p? 1, 2 What is q? 1, 3

List of **POSSIBLE** roots: $\frac{1}{1}, \frac{2}{1}, \frac{1}{3}, \frac{2}{3} \rightarrow \boxed{\pm 1, \pm 2, \pm \frac{1}{3}, \pm \frac{2}{3}}$

EX 3: Find the possible rational roots of $2x^3 + 5x^2 - 6x - 8 = 0$.

8 2

What is p? 1, 2, 4, 8 What is q? 1, 2

List of **POSSIBLE** roots: $\frac{1}{1}, \frac{2}{1}, \frac{4}{1}, \frac{8}{1}, \frac{1}{2}, \frac{2}{2}, \frac{4}{2}, \frac{8}{2}$

$\boxed{\pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{2}}$

Steps for Finding the Zeros of a Polynomial Function:

1) Gather General Information.
   - Determine the **degree** ***n*** of the polynomial function.
   - The **number of zeros** of the polynomial function is **at MOST *n (the degree)*.**
2) Check rational zeros.
   - Apply the **Rational Zero Theorem** to list rational numbers that are ***POSSIBLE* zeros**.
   - Use **synthetic division** to test the numbers in the list.
     - The number is a solution if the remainder is **ZERO.**
3) Work with the **reduced/depressed** polynomial.
   - Each time a zero is found, obtain the reduced/depressed polynomial.
   - Work to get a **reduced polynomial of degree 2.**
   - Then, find its zeros by ***factoring*** or by applying the ***quadratic formula***.

EX 4: Find the zeros of $f(x) = x^3 - 7x^2 + 16x - 12$.

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At most 3 zeros.

Rational Root Theorem – Possible rational zeros:

$\frac{12}{1} \rightarrow \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

Find the roots:

$$\begin{array}{r|rrrr} 2 & 1 & -7 & 16 & -12 \\ & \downarrow & 2 & -10 & 12 \\ \hline & 1 & -5 & 6 & \boxed{0} \end{array}$$

$x^2 - 5x + 6 = 0$

$(x-2)(x-3) = 0$

$\boxed{x = 2 \quad x = 2 \quad x = 3}$

EX 5: Find the zeros of $g(x) = 3x^4 + 23x^3 + 56x^2 + 52x + 16$

At most 4 zeros.

Rational Root Theorem – Possible rational zeros:

16: 1, 2, 4, 8, 16

3: 1, 3

$\boxed{\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{8}{3}, \pm \frac{16}{3}}$

Find the roots:

$$\begin{array}{r|rrrrr} -1 & 3 & 23 & 56 & 52 & 16 \\ & \downarrow & -3 & -20 & -36 & -16 \\ \hline & 3 & 20 & 36 & 16 & \boxed{0} \end{array} \rightarrow 3x^3 + 20x^2 + 36x + 16$$

$$\begin{array}{r|rrrr} -2 & 3 & 20 & 36 & 16 \\ & \downarrow & -6 & -28 & -16 \\ \hline & 3 & 14 & 8 & \boxed{0} \end{array} \rightarrow 3x^2 + 14x + 8 = 0$$

$$\begin{array}{r|rrr} -4 & 3 & 14 & 8 \\ & \downarrow & -12 & -8 \\ \hline & 3 & 2 & \boxed{0} \end{array}$$

$3x + 2 = 0$

$(3x+2)(x+4) = 0$

2x

12x

$\boxed{x = -1 \quad x = -2 \quad x = -4 \quad x = -\frac{2}{3}}$

When a root occurs more than once, we say that root has
MULTIPLICITY

Example: State the zeros and multiplicity for the polynomial function.

$f(x)=(x+2)(x+2)(x+2)$	$f(x)=x^2(x-3)^4(x+2)$
$x+2=0$ $x+2=0$ $x+2=0$ $x=-2$ $x=-2$ $x=-2$ $x=-2$ mult. of 3	$x=0$ mult. of 2 $x=-2$ mult. of 1 $x=3$ mult. of 4 4 times $x=0$ $x=0$ $x-3=0$ $x+2=0$ $x^2=x\cdot x$ $x=3$

Around the room Practice

-Yes, you will turn in.

-Yes, you need to show work

-No, your order will not be the same as others.

$\frac{1}{4} = \frac{1}{1,2,4} = \frac{1}{1}\ \frac{1}{2}\ \frac{1}{4}$

Quick Check Tomorrow!