Welcome! Happy Monday!

-Cell phones in holders.

Unit 3 Polynomials Functions

Important Dates:

Unit Quiz 1 Friday 9/13

Unit Quiz 2 Friday 9/20

Remainder Theorem and Factor Theorem

What is the difference between a factor and a solution?

factor - mult. to get poly. $(x \pm \#)$

solution - a # that makes a true statement $(x = \#)$

What is the difference between a solution and a root?

Nothing

root = solution

Given the factor find the solution:

1. $(x - 7)$

$x = 7$

2. $(3x+1)$

$x = -\frac{1}{3}$

$3x + 1 = 0$
$-1 \quad -1$
$3x = -1$
$x = -1/3$

Given the root find the factor:

3. $x = 4$

$(x-4)$

4. $x = \frac{1}{2}$

$(2x-1)$

$2 \cdot x = \frac{1}{2} \cdot 2$

$2x = 1$

$2x - 1 = 0$

Remainder Theorem:

The remainder from synthetic division is the value of the function at that point.

Example: Use synthetic division to find the value of P(5) if
$P(x) = 3x^4 - 5x^2 + x - 9$.

The value for x is 5 because of the function notation. Put 5 outside the box and put the coefficients of the polynomial inside the box. Remember to use **all** descending powers, so there's a $0x^3$.

5	3	0	−5	1	−9
		15	75	350	1755
	3	15	70	351	1746

The remainder is 1746, so P(5)=1746.

Let's Try:

3. Use synthetic division to find the value of P(-3) if
$P(x) = x^4 - 5x^2 + x + 10$.

x = -3

-3	1	0	-5	1	10
	↓	-3	9	-12	+33
	1	-3	4	-11	43

P(-3) = 43

4. Use synthetic division to evaluate P(2) if
$P(x) = x^3 - 12x^2 + 8x + 9$.

2	1	-12	8	9
	↓	2	-20	-24
	1	-10	-12	-15

P(2) = -15

Factor Theorem:

If a remainder is **zero**, the number outside the box in synthetic division is a zero of the function; therefore, the related factor is a factor of the polynomial.

Example: Given $f(x) = 2x^3 + 11x^2 + 18x + 9$, is (x+3) a factor of the polynomial?

Because they give you a factor, you must set equal to zero and solve for the solution. Put -3 outside the box, and the coefficients of the polynomial inside the box.

$$\begin{array}{r|rrrr} -3 & 2 & 11 & 18 & 9 \\ & & -6 & -15 & -9 \\ \hline & 2 & 5 & 3 & 0 \end{array}$$

The remainder is 0, so x = -3 is a zero of the function. That means (x + 3) is a factor of the polynomial

$2x^2+5x+3$

$(2x+3)(x+1)$

Factor

$(2x+3)(x+1)(x+3)$

Let's try:

5. Given $f(x) = 2x^3 + 11x^2 + 18x + 9$, is (x - 1) a factor of the polynomial?

$x-1=0$

$x=1$

$$\begin{array}{r|rrrr} 1 & 2 & 11 & 18 & 9 \\ & \downarrow & 2 & 13 & 31 \\ \hline & 2 & 13 & 31 & 40 \end{array}$$

NO

6. Given $f(x) = x^4 + 12x^2 + 18x + 9$, is (x + 3) a factor of the polynomial?

$x+3=0$

$x=-3$

$$\begin{array}{r|rrrrr} -3 & 1 & 0 & 12 & 18 & 9 \\ & \downarrow & -3 & 9 & -63 & 135 \\ \hline & 1 & -3 & 21 & -45 & 144 \end{array}$$

NO

7. Determine the value of "d" if x = 3 is a zero of the polynomial $-4x^3 + 5x^2 + 2x + d$.

$$\begin{array}{r|rrrr} 3 & -4 & 5 & 2 & d \\ & \downarrow & -12 & -21 & -57 \\ \hline & -4 & -7 & -19 & d-57 \end{array}$$

$d-57=0$

$d=57$

The Rational Zero Theorem:

WHY IT IS IMPORTANT: <u>Narrows the search for rational zeros to a finite list.</u>

- If $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ has integer coefficients $(a_n \neq 0)$ and $\frac{p}{q}$ is a rational zero (in lowest terms) of p, then **p is a factor of the constant term a_0 and q is a factor of the leading coefficient a_n.**

EX 1: Find the roots of $x^3 + 6x^2 + 10x + 3 = 0$.

HINT: Apply the **Rational Root Theorem** to find the **possible** rational roots!

What is p? ________

What is q? ________

EX 2: Find the possible rational roots of $3x^3 + 5x^2 + 7x + 2 = 0$.

What is p? ________

What is q? ________

EX 3: Find the possible rational roots of $2x^3 + 5x^2 - 6x - 8 = 0$.

What is p? ________

What is q? ________