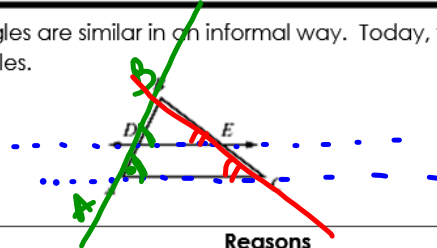


Day 3 – Proving Theorems about Similarity

From yesterday, you learned how to determine if two triangles are similar in an informal way. Today, you will prove several different theorems about similarity with triangles.

Given: $\overline{AC} \parallel \overline{DE}$
 Prove: \overline{DE} divides \overline{AB} and \overline{CB} proportionally.



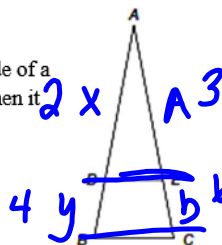
Statements	Reasons
1. $\overline{AC} \parallel \overline{DE}$	1. Given
2. $\angle BDE \cong \angle BAC$	2. Corresponding angles are congruent
3. $\angle BED \cong \angle BCA$	3. Corresponding \angle s are \cong
4. $\triangle BDE \sim \triangle BAC$	4. AA
5. $\frac{BD}{BA} = \frac{BE}{BC}$	5. Corresponding sides of similar Δ s are proportional
6. $\overline{BD} + \overline{DA} = \overline{BA}$	6. Segment Addition
7. $\overline{BE} + \overline{EC} = \overline{BC}$	7. Segment Addition
8. $\frac{BD}{\overline{BD} + \overline{DA}} = \frac{BE}{\overline{BE} + \overline{EC}}$	8. Substitution
9. $BE(\overline{BD} + \overline{DA}) = \overline{BD}(\overline{BE} + \overline{EC})$	9. Rewrite
10. $\overline{BE} \cdot \overline{BD} + \overline{BE} \cdot \overline{DA} = \overline{BD} \cdot \overline{BE} + \overline{BD} \cdot \overline{EC}$	10. Simplify
11. $\overline{BE} \cdot \overline{DA} = \overline{BD} \cdot \overline{EC}$	11. Subtractions.
12. \overline{DE} divides \overline{AB} and \overline{CB} proportionally.	12. Def. of proportional

Side Splitter Theorem (Triangle Proportionality Theorem)

The **Side Splitter Theorem**, which states "If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally."

If: $\overline{DE} \parallel \overline{BC}$

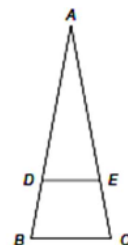
Then: $\frac{AD}{DB} = \frac{AE}{EC}$

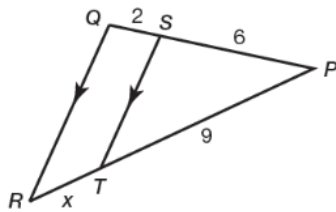


The **Converse of the Side Splitter Theorem**, states "If a line divides two sides of a triangle proportionally, then it is parallel to the third side."

If: $\frac{AD}{DB} = \frac{AE}{EC}$

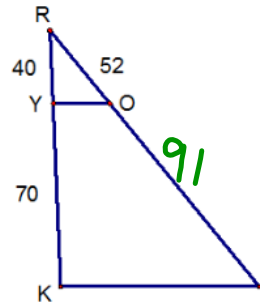
Then: $\overline{DE} \parallel \overline{BC}$



Example 1: Find the value of x if $ST \parallel QR$.

$$\frac{6}{2} = \frac{9}{x}$$

$$x = 3$$

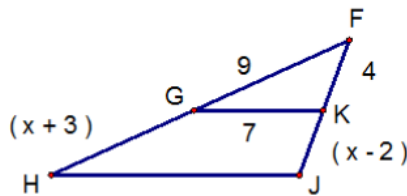
Example 2: Find RC if $YO \parallel KC$.

$$\frac{40}{70} = \frac{52}{OC}$$

$$40(OC) = 3640$$

$$OC = 91$$

$$RC = 52 + 91 \quad RC = 143$$

Example 3: Find the value of x if $GK \parallel HJ$.

$$\frac{9}{x+3} = \frac{4}{x-2}$$

$$4(x+3) = 9(x-2)$$

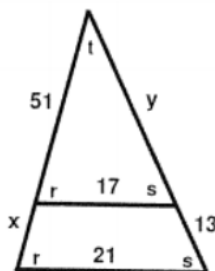
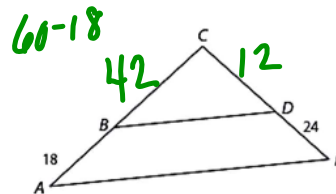
$$4x + 12 = 9x - 18$$

$$-4x \quad -4x$$

$$\frac{12}{12} = \frac{5x - 18}{12}$$

$$30 = 5x$$

$$6 = x$$

Example 5: Find x and y :**Example 4:** If $AC = 60$ units and $EC = 36$ units, is $\overline{AE} \parallel \overline{BD}$?

$$\frac{12}{24} = \frac{42}{18}$$

$$\frac{1}{2} \neq \frac{7}{3}$$

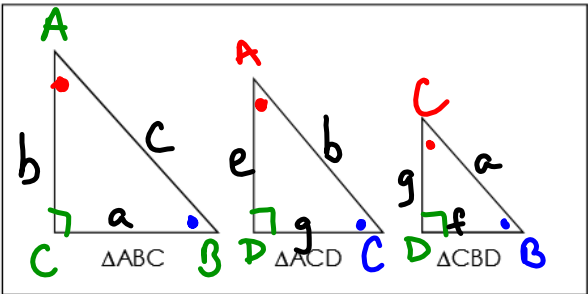
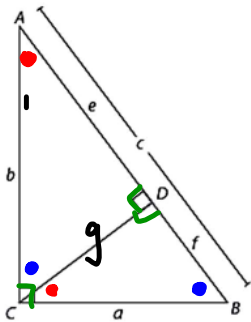
No \overline{AE} is not \parallel to \overline{BD}

Proving the Pythagorean Theorem Using Similar Triangles

Prove the Pythagorean Theorem using Similar Triangles.

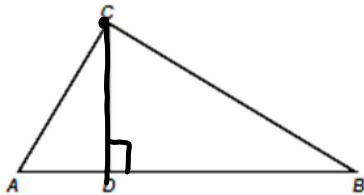
Given: Triangle ABC with $\angle C$ being a right angle
 \overline{CD} is an altitude

Prove: $a^2 + b^2 = c^2$



Statements	Reasons
1. $\frac{b}{c} = \frac{e}{b}$ and $\frac{a}{c} = \frac{f}{a}$	1.
2. $b^2 = ec$ and $a^2 = cf$	2. Cross Multiply
3.	3. Addition
4. $a^2 + b^2 = c(e + f)$	4.
5.	5. Substitution
6.	6. Multiply

From the proof above, we developed a relationship between the three triangles.



If: $\triangle ABC \sim \triangle ACD \sim \triangle CBD$,

Then:

$\frac{BD}{CD} = \frac{CD}{AD}$ and $\frac{AB}{BC} = \frac{BC}{BD}$ and $\frac{AB}{AC} = \frac{AC}{AD}$

OR

$\frac{\text{right}}{x} = \frac{x}{\text{left}}$ and $\frac{\text{whole}}{y} = \frac{y}{\text{right}}$ and $\frac{\text{whole}}{z} = \frac{z}{\text{left}}$

A right triangle with vertices A, B, and N. The right angle is at vertex A, indicated by a green square. Vertex B is at the top left, and vertex N is at the bottom right. An altitude is drawn from vertex A to the hypotenuse BN, meeting it at point T. This altitude is perpendicular to BN, indicated by a green square at T. This construction creates three right triangles: $\triangle ABT$, $\triangle ANT$, and $\triangle BNT$. The triangles $\triangle ABT$ and $\triangle ANT$ are shaded in light blue. A red dot is located on the segment AN.

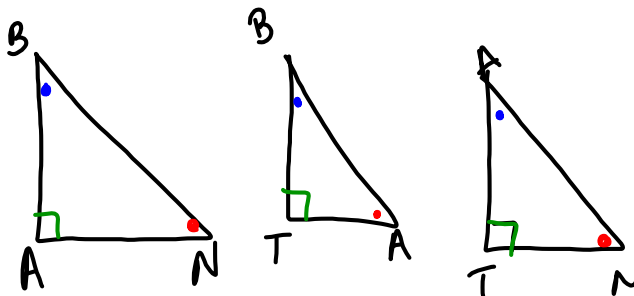


Diagram illustrating the solution for the first problem using similar triangles. A large triangle ABC is shown with altitude CD drawn from vertex C to the base AB . The base AB is divided into segments $AD = 18$ and $DB = 10$. The altitude CD is labeled x . A boxed equation states $x = 6\sqrt{5}$. Below the main diagram, three smaller right triangles are shown, each with a right angle symbol at vertex C or D :

- Triangle ACD (left): Right angle at C , side $AD = 18$, hypotenuse $AC = 28$.
- Triangle BCD (middle): Right angle at D , side $DB = 10$, hypotenuse $BC = x$.
- Triangle ABC (right): Right angle at C , side $AC = 18$, hypotenuse $AB = x$.

The similarity relationships are indicated by the boxed equations:

$$\frac{x}{18} = \frac{18}{x}$$

Example 10: Find the value of x.

Handwritten calculations:

$$\frac{4}{x} = \frac{x}{12}$$

$$\sqrt{x^2 = 48}$$

$$x = 4\sqrt{3}$$

$\frac{10}{x} = \frac{x+21}{10}$

$$x(x+2) = 10 \cdot 10$$

$$x^2 + 21x = 100$$

$$\begin{aligned} x^2 + 21x - 100 &= 0 \\ \text{Factor} \\ (x+25)(x-4) &= 0 \end{aligned}$$

$x=4$