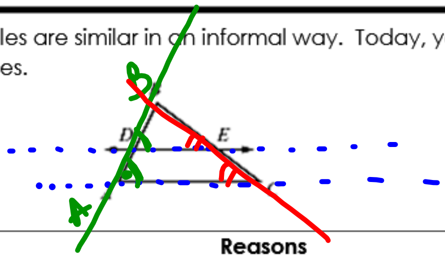
Day 3 – Proving Theorems about Similarity

From yesterday, you learned how to determine if two triangles are similar in an informal way. Today, you will prove several different theorems about similarity with triangles.



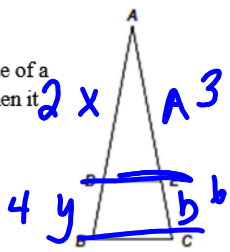
Given: Given: $\overline{AC} \parallel \overline{DE}$

Prove: $\overleftrightarrow{DE}$ divides $\overline{AB}$ and $\overline{CB}$ proportionally.

Statements	Reasons
1. $\overline{AC} \parallel \overline{DE}$	1. Given
2. $\angle BDE \cong \angle BAC$	2. Corresponding angles are congruent
3. $\angle BED \cong \angle BCA$	3. Corresponding $\angle$s are $\cong$
4. $\triangle BDE \sim \triangle BAC$	4. AA
5. $\frac{BD}{BA} = \frac{BE}{BC}$	5. Corresponding sides of similar Δs are proportional
6. $\overline{BD} + \overline{DA} = \overline{BA}$	6. Segment Addition
7. $\overline{BE} + \overline{EC} = \overline{BC}$	7. Segment Addition
8. $\frac{BD}{BD+DA} = \frac{BE}{BE+EC}$	8. Substitution
9. $\overline{BE}(\overline{BD}+\overline{DA}) = \overline{BD}(\overline{BE}+\overline{EC})$	9. Rewrite
10. $\overline{BE}\cdot\overline{BD} + \overline{BE}\cdot\overline{DA} = \overline{BD}\cdot\overline{BE} + \overline{BD}\cdot\overline{EC}$	10. Simplify
11. $\overline{BE}\cdot\overline{DA} = \overline{BD}\cdot\overline{EC}$	11. Subtractions.
12. $\overleftrightarrow{DE}$ divides $\overline{AB}$ and $\overline{CB}$ proportionally.	12. Def. of proportional

Side Splitter Theorem (Triangle Proportionality Theorem)

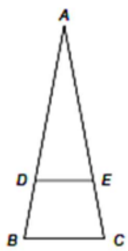
The **Side Splitter Theorem**, which states "If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally."



If: $\overline{DE} \parallel \overline{BC}$

Then: $\frac{AD}{DB} = \frac{AE}{EC}$

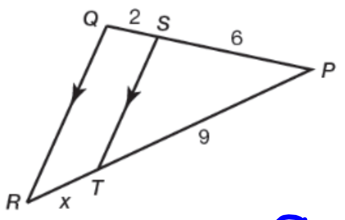
The **Converse of the Side Splitter Theorem**, states "If a lines divides two sides of a triangle proportionally, then it is parallel to the third side."



If: $\frac{AD}{DB} = \frac{AE}{EC}$

Then: $\overline{DE} \parallel \overline{BC}$

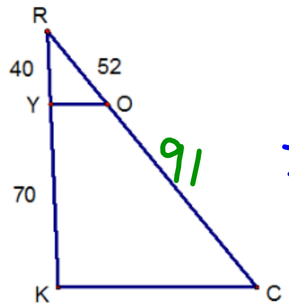
Example 1: Find the value of x if ST || QR.



$$\frac{6}{2} = \frac{9}{x}$$

$$x = 3$$

Example 2: Find RC if YO || KC.



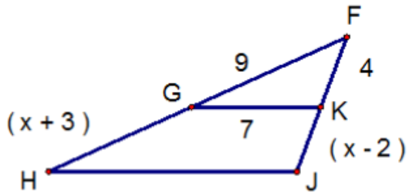
$$\frac{40}{70} = \frac{52}{OC}$$

$$40(OC) = 3640$$

$$OC = 91$$

$$RC = 52 + 91 \qquad RC = 143$$

Example 3: Find the value of x if GK || HJ.



$$\frac{9}{x+3} = \frac{4}{x-2}$$

$$4(x+3) = 9(x-2)$$

$$4x + 12 = 9x - 18$$

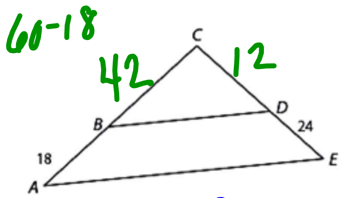
$$-4x \qquad -4x$$

$$12 = 5x - 18$$

$$30 = 5x$$

$$6 = x$$

Example 4: If AC = 60 units and EC = 36 units, is $\overline{AE} \parallel \overline{BD}$?

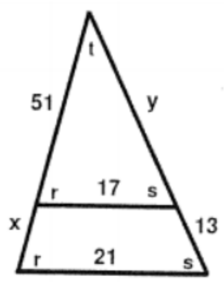


$$\frac{12}{24} = \frac{42}{18}$$

$$\frac{1}{2} \neq \frac{7}{3}$$

No $\overline{AE}$ is not // to $\overline{BD}$

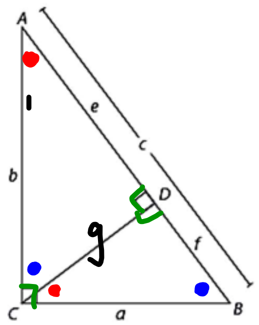
Example 5: Find x and y:

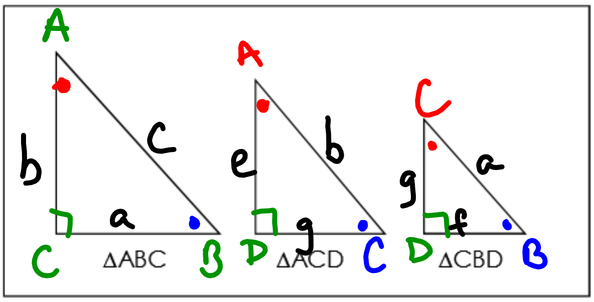


Proving the Pythagorean Theorem Using Similar Triangles

Prove the Pythagorean Theorem using Similar Triangles.
Given: Triangle ABC with ∠C being a right angle
$\overline{CD}$ is an altitude

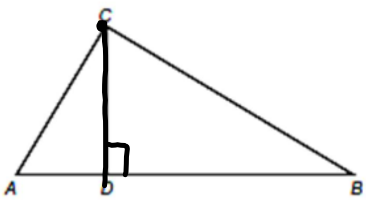
Prove: $a^2 + b^2 = c^2$





Statements	Reasons
1. $\frac{b}{c} = \frac{e}{b}$ and $\frac{a}{f} = \frac{c}{a}$	1.
2. $b^2 = ec$ and $a^2 = cf$	2. Cross Multiply
3.	3. Addition
4. $a^2 + b^2 = c\,(e + f)$	4.
5.	5. Substitution
6.	6. Multiply

From the proof above, we developed a relationship between the three triangles.



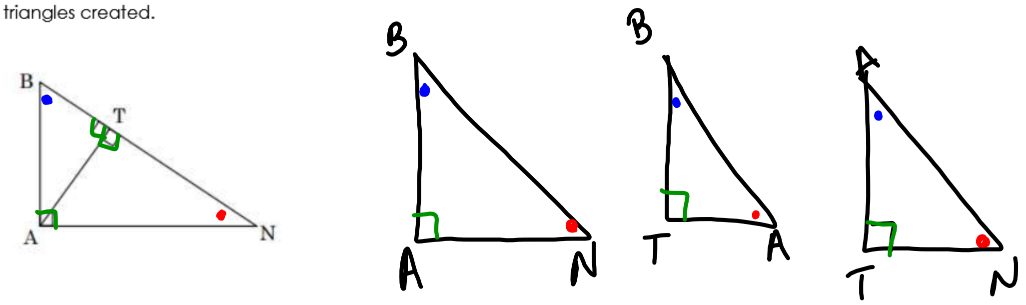
If: ΔABC ~ ΔACD ~ ΔCBD ,
Then:

$$\frac{BD}{CD} = \frac{CD}{AD} \text{ and } \frac{AB}{BC} = \frac{BC}{BD} \text{ and } \frac{AB}{AC} = \frac{AC}{AD}$$

OR

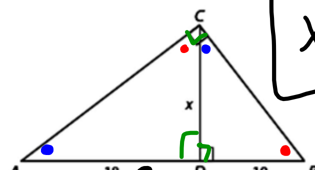
$$\frac{\text{right}}{x} = \frac{x}{\text{left}} \text{ and } \frac{\text{whole}}{y} = \frac{y}{\text{right}} \text{ and } \frac{\text{whole}}{z} = \frac{z}{\text{left}}$$

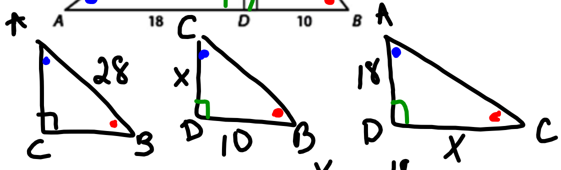
Example 6: If an altitude is drawn to the hypotenuse of triangle BAN below, then name and redraw the 3 similar triangles created.



Example 7: Find the value of x:

$\boxed{x = 6\sqrt{5}}$

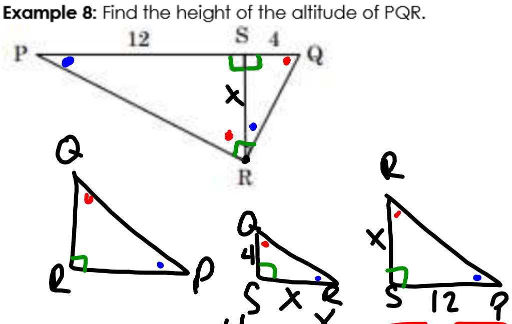




$$\frac{x}{10} = \frac{18}{x}$$

$$\sqrt{x^2} = \sqrt{180}$$

Example 8: Find the height of the altitude of PQR.

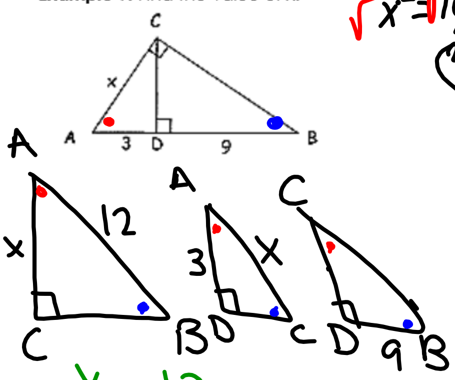


$$\frac{4}{x} = \frac{x}{12}$$

$$\sqrt{x^2} = \sqrt{48}$$

$\boxed{x = 4\sqrt{3}}$

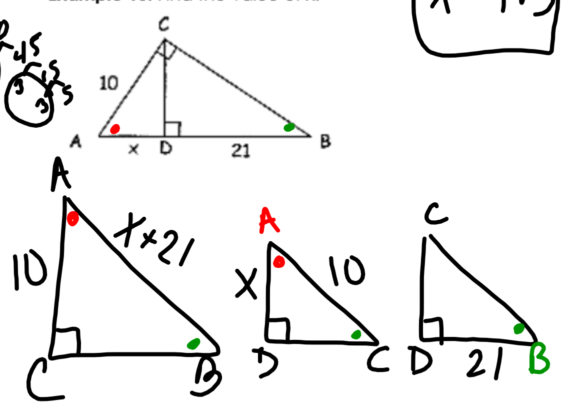
Example 9: Find the value of x.



$$\frac{x}{3} = \frac{12}{x}$$

$\boxed{x = 6}$

Example 10: Find the value of x.



$$\frac{10}{x} = \frac{x+21}{10}$$

$$x(x+21) = 10 \cdot 10$$

$$x^2 + 21x = 100$$

$$-100 \quad -100$$

$$x^2 + 21x - 100 = 0$$

Factor

$$(x+25)(x-4) = 0$$

$x = 4$