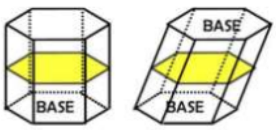
Volume (Pyramids & Spheres)

Volume is amount of space contained in an object or the number of unit cubes of a given size that will exactly fill the interior of a three dimensional figure. We will learn volume formulas for 2 different 3-D objects today.

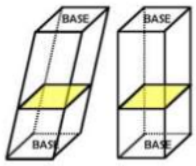
Cavalieri's Principle

Cavalieri's Principle states that if two three dimensional figures have the same height and the same cross sectional area at every level, they have the same volume. In other words, if two figures have the same dimensions (height, radii, base, etc), but are just slanted or oblique, they will have the same volume. Each of the figures above would have the same volume because they have the same height and cross section, even though they are slanted.

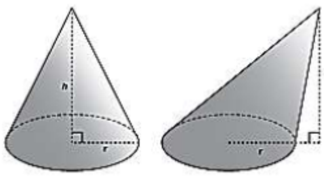
Examples of two figures with the same volume:



Volumes are equal.

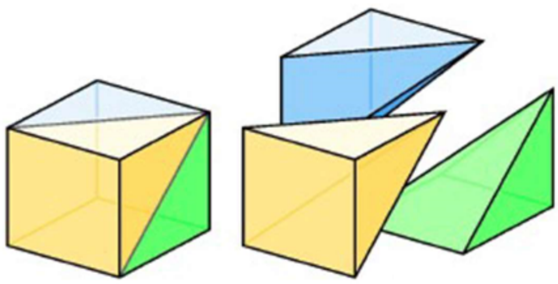


Volumes are equal.



Discovering the Volume of a Pyramid

A **prism** is a solid object with, identical ends, flat rectangular faces and bases, and the same cross section all along its length. A **pyramid** is a solid object that has a base and three or more triangular faces that meet at a point above the base. *A square prism and a rectangular prism are made up of three pyramids of equal volume.* The volume of a prism is V = Bh, where B is the area of the base and h is the height of the prism.



Thus if I told you the volume of the above cube (a square prism) is 51 m^3, what would you tell me is the volume of one of the pyramids that make up the cube? 1/3

Find the following using the same logic:

1. Volume of square prism = 126 in³ Volume of pyramid = 42 in^3
2. Volume of square prism = 216 ft³ Volume of pyramid = 72 ft^3
3. Volume of square prism = 87 m^3 Volume of pyramid = 29m³

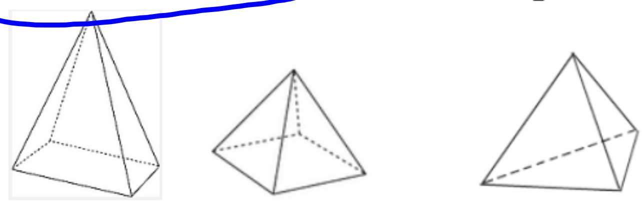
Using the information given above and our calculations, we can conclude that the volume of a pyramid is:

Volume of a Pyramid* = $\frac{1}{3} \cdot B \cdot h$ * B is the area of Base

*Considering that a pyramid can have multiple bases, whatever shape the base is you will replace B with the formula for that shape.

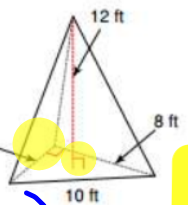
Possible Base Formulas

Rectangle/Square: $A = lw$ **Triangle:** $A = \frac{bh}{2}$ or $\frac{1}{2}bh$



Practice with Pyramids

A. Find the volume of the following pyramids.



option 1

$\frac{1}{3}\left(\frac{1}{2} \cdot 8 \cdot 6\right) \cdot 12$

$\boxed{96 ft^3}$

Triangle

$b = 8$

$h = 6$

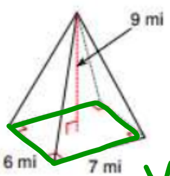
$B = \frac{1}{2} b h$

$B = \frac{1}{2}(8)(6)$

$B = 24$

$Vol = \frac{1}{3} B \cdot h$

$V = \frac{1}{3}(24)(12)$ $V = 96$



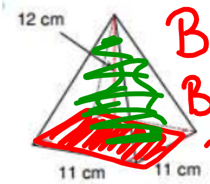
Rect

$B = (6)(7)$

$B = 42$

$V = \frac{1}{3}(42)(9)$

$\boxed{V = 126 mi^3}$



Area of sq.

$B = LW$

$B = 11 \cdot 11$

$B = 121$

$V = \frac{1}{3} B h$

$V = \frac{1}{3}(121)(12)$

$V = 484 cm^3$

B. Find the height of a rectangular pyramid with a length 3 m, width 8 m, and a volume of 112 m³.

$V = \frac{1}{3} B h$

$V = \frac{1}{3} \cdot L \cdot W \cdot h$

$112 = \boxed{\frac{1}{3} \cdot 3 \cdot 8} h$

$\frac{112}{8} = \frac{8h}{8}$

$\boxed{h = 14 m}$

C. The area of the base of a square pyramid is 7582 square feet. What is the length of the base?

$A = L \cdot W$

$7582 = x \cdot x$

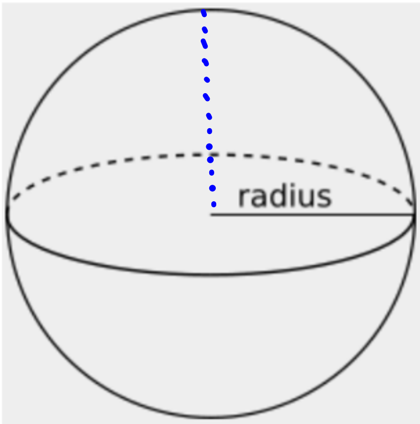
$7582 = x^2$

$87.1 ft = x$

Volume of a Sphere

In order to discover the volume of a sphere, we would have to use Cavalieri's Principle, the volume of a cone, and the volume of a cylinder (which we will be learning tomorrow). Thus, we don't have enough information to discover it just yet. The volume of a sphere is:

Volume of a Sphere = $4/3\pi r^3$



Where r is the radius.

Practice with Spheres

A. Find the volume of the spheres. Write your answers exactly and approximately.

$V = \frac{4}{3}\pi \cdot (22)^3$

$V = 44,602.24 \text{ cm}^3$

$V = 14,197.33\pi \text{ cm}^3$

$V = \frac{4}{3}\pi (9.1)^3$

$V \approx 3,156.55$

$V = 1004.76\pi$

B. A rubber ball has a radius of 30 cm. What is the volume of the ball?

$V = 36000\pi \text{ cm}^3$

$V \approx 113{,}097.34 \text{ cm}^3$

C. Find the diameter of a sphere with a volume of 972π in³.

$V = \frac{4}{3}\pi r^3$

$972\pi = \frac{4}{3}\pi r^3$

$\frac{3}{4} \cdot 972 = \frac{4}{3} r^3 \cdot \frac{3}{4}$

$729 = r^3$

$9 = r$

$\boxed{d = 18 \text{ in}}$

D. Given that the volume of a sphere is 5276 cm³, find its radius.

$\frac{3}{4} \cdot 5276 = \frac{4}{3}\pi r^3 \cdot \frac{3}{4}$

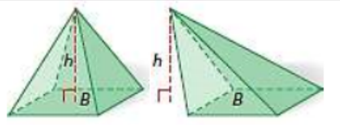
$\frac{3957}{\pi} = \frac{\pi r^3}{\pi}$

$1259.56 = r^3$

$\boxed{10.8 = r}$

Putting it all Together

We discovered the volume of a pyramid and were given the volume of a sphere, below is a table with the formulas for both of the 3-D objects:

	
Pyramid $V=1/3Bh$ Where *B* is area of the base and *h* is the height *Whatever shape the base is you will replace B with the formula for that shape.	**Sphere** $V=4/3\pi r^3$ Where r is the radius

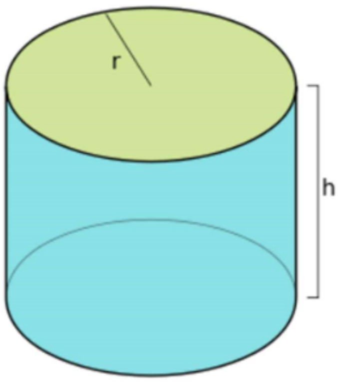
Volume (Cylinders and Cones)

Yesterday, we learned about volume, specifically the volume of a pyramid and sphere. Today, we will learn the volume formulas of 2 more 3-D objects: cylinders and cones.

Discovering the Volume of a Cylinder

One thing we learned yesterday was the importance of the area of the base. What is the base of EVERY cylinder? Circle

What is the area of the base of a cylinder? (area of the shape you listed above) $A = \pi r^2$

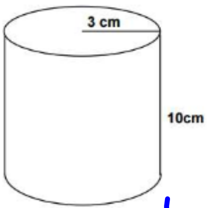


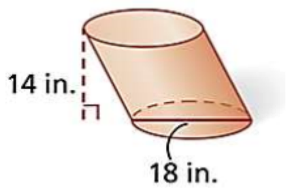
The information that we have only covers the base. Using the image above and the information you have, what do you conclude is the formula for the volume of a cylinder?

Volume of a Cylinder = $\pi r^2 h$

Practice with Cylinders

A. Find the volume. Write your answer exactly and approximately.





$V = \pi r^2 h$

$V = \pi (3)^2 \cdot 10$

$V = 90\pi \text{ cm}^3$

$V \approx 282.7 \text{ cm}^3$

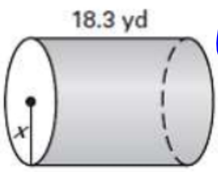
$V = 1134\pi \text{ in}^3$

$V \approx 3562.57 \text{ in}^3$

$V = \pi r^2 h$

B. Solve for x.

$V = 3148\ \text{yd}^3$



$3148 = \pi \cdot r^2 \cdot 18.3$

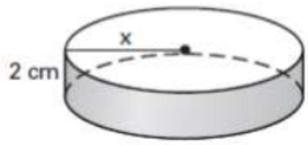
$\frac{3148}{(18.3\pi)} = \frac{\pi r^2 18.3}{18.3\pi}$

$54.76 = r^2$

$\boxed{7.4\ \text{yd} = r}$

$3148 \div (18.3\pi)$

$V = 72\pi\ \text{cm}^3$



$\boxed{x = 6\ \text{cm}}$

C. Find the volume of a cylinder with base area 121π cm² and a height equal to twice the radius. Give your answer in terms of π and rounded to the nearest tenth.

$A = \pi r^2$

$121\pi = \pi r^2$

$121 = r^2$

$11 = r$

$h = 2r$

$h = 2 \cdot 11$

$h = 22$

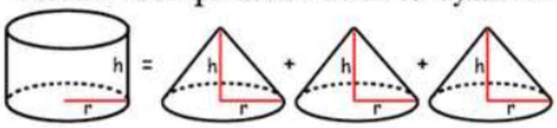
$V = \pi (11)^2 \cdot 22$

$V = 2662\pi\ \text{cm}^3$

$V \approx 8362.92\ \text{cm}^3$

Discovering the Volume of a Cone

Volume Comparison: Cone & Cylinder



Above is a volume comparison between cylinders and cones. The volume of three cones is the equivalent to the volume of one cylinder or it takes three cones to fill one cylinder.

Find the following using the information above:

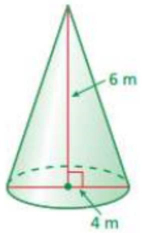
4. Volume of a cylinder = 36 in³ Volume of a cone = 12 in³
5. Volume of a cylinder = 96 ft³ Volume of a cone = 32 ft³
6. Volume of a cylinder = 132 m³ Volume of a cone = 44 m³

Using the information given above and our calculations, we can conclude that the volume of a cone is:

Volume of a Cone = $\frac{1}{3}\pi r^2 h$

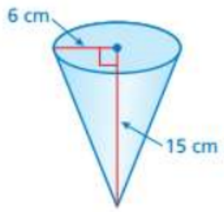
Practice with Cones

A. Find the volume of the cone. Write your final answers exactly and approximately.



$V = 8\pi\ m^3$

$V \approx 25.13 m^3$



$V = 180\pi cm^3$

$V \approx 565.49 cm^3$

B. Find the missing dimension given the volume.

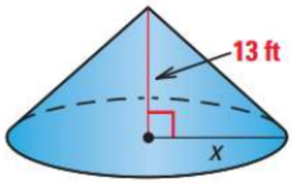
Volume = 225 cm^3

$225 = \frac{1}{3}\pi(5)^2 \cdot h$

$225 = 26.18h$

$\frac{225}{26.18} = \frac{26.18h}{26.18}$

$\boxed{8.6 cm = h}$



Volume = 2614 ft^3

$2614 = \frac{1}{3} \cdot \pi \cdot r^2 \cdot 13$

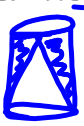
$2614 = 13.61 r^2$

$\frac{2614}{13.61} = \frac{13.61r^2}{13.61}$

$\sqrt{192.01} = \sqrt{r^2}$

$13.9 ft = r$

C. A cone just fits inside a cylinder with volume 300 cm³. What is the volume of the cone?



$V = \frac{1}{3}\pi r^2 \cdot h$ $V = \pi r^2 \cdot h$

$V = \frac{300}{3}$ $\boxed{V = 100 cm^3}$

A cone has a volume of 150 cm³. What is the volume of a cylinder that just holds the cone?

$\boxed{V = 450 cm^3}$

D. Find the volume of a cone with base circumference 25π in. and a height 2 in. more than twice the radius.

$C = 2\pi r$

$25\pi = 2\pi r$

$25 = 2r$

$12.5 = r$

$h = 2r + 2$

$h = 2(12.5) + 2$

$h = 27$

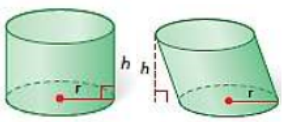
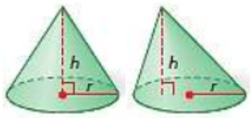
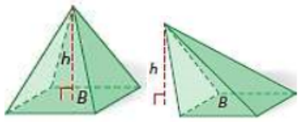
$V = \frac{1}{3} \cdot \pi \cdot r^2 \cdot h$

$V = \frac{1}{3} \cdot \pi \cdot (12.5)^2 (27)$

$\boxed{V = 4417.86 in^3}$

Putting it all Together

We can now add two more shapes to our table, volume of a sphere and volume of a cone. Below is a table with the formulas for all of the 3-D objects:

Volumes			
			
Cylinders *V=Bh* where *B* is area of the base and *h* is the height V=$\pi r^2 h$ πr^2 represents the area of the base (circle)	**Cone** *V=1/3Bh* where *B* is area of the base and *h* is the height V=$1/3\pi r^2 h$ πr^2 represents the area of the base (circle)	**Pyramid** *V=1/3Bh* Where *B* is area of the base and *h* is the height *Whatever shape the base is you will replace B with the formula for that shape.	**Sphere** V=4/3πr^3 Where r is the radius

Application of Volume

1. A standard size sheet of paper measures 8.5 inches by 11 inches. Use two standard sized sheets of paper to create two cylinders. One cylinder should have a height of 11 inches and the other cylinder should have a height of 8.5 inches. Predict which cylinder will have the greatest volume: ____________________________

a. Determine the radius of each cylinder:

h = 11 inches

r = 4.25

h = 8.5 inches

r = 5.5

b. Calculate the volume of each cylinder.

H = 11 inches

V = 624.2

h = 8.5 inches

V = 807.8

c. Does the radius or height have a greater impact on the magnitude of the volume? Why?

b/c You are Squaring radius

d. What does the radius need to be on the cylinder with a height of 11 inches so the volume of the cylinder with a height of 11 inches equals the volume of the cylinder with a height of 8 inches?

S & P

1, 2, 3, 7

C & C

1, 5, 10