Try the problems below

1. In the diagram below of triangle CDE, F is a midpoint of $\overline{CD}$ and G is a midpoint of $\overline{DE}$. If $FG = 4x + 12$, and $CE = 4x + 28$, what is the measure of CE?

midseg = $\frac{1}{2}$ // side

// side = 2 · midsegment

$4x+28 = 2(4x+12)$

$4x+28 = 8x+24$

$4x+12 = \frac{1}{2}(4x+28)$

$4x+12 = 2x+14$

$-2x \quad -2x$

$2x+12 = 14$

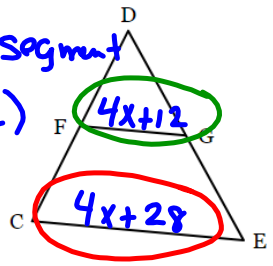
$2x = 2$

$x = 1$

$CE = 4x+28$

$CE = 4(1)+28$

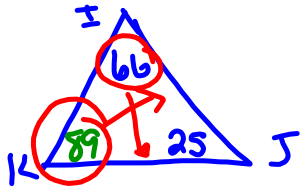
$CE = 32$



2. In triangle IJK, m< I = 66 and m<J= 25. List the side of trianlge IJK in order from longest to shortest.

$66+25+m\angle K = 180$

$\overline{IJ}, \overline{KJ}, \overline{IK}$



3. In ΔQRS, $m\angle Q = (7x + 7)^\circ$, $m\angle R = (4x + 14)^\circ$, and $m\angle S = (x + 15)^\circ$. Find $m\angle S$.

$7x+7+4x+14+x+15 = 180$

$12x+36 = 180$

$-36 \quad -36$

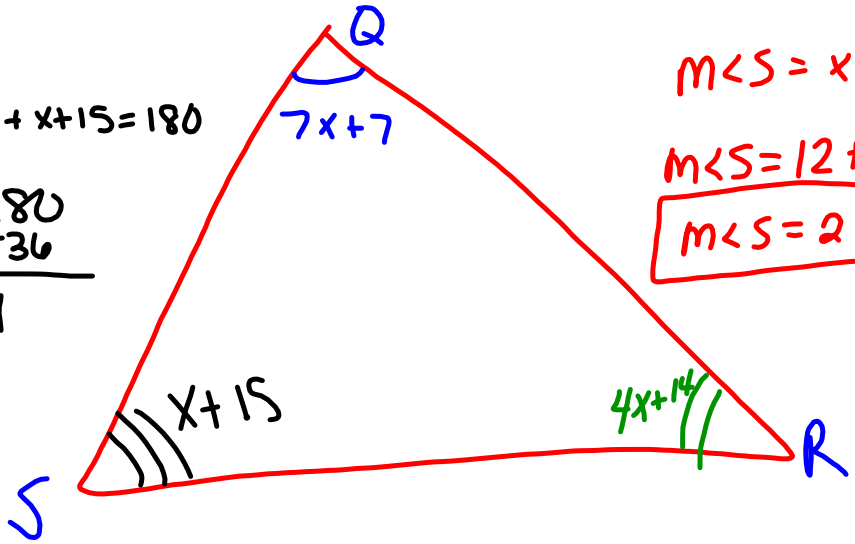
$12x = 144$

$x = 12$

$m\angle S = x+15$

$m\angle S = 12+15$

$m\angle S = 27$



Touchstone Quick Check

-Please complete the touchstone and place answers in the remote.

For question 4 the answer is C

Geometry Unit 2: Triangle Congruence and Quadrilaterals Notes

Properties of Quadrilaterals

A **parallelogram** is a type of quadrilateral that has **two pairs of opposite sides that are parallel.** Parallelograms are denoted by the symbol ▱. If a quadrilateral has two pairs of parallel, opposite sides, then it can be classified as a parallelogram.

There are 5 theorems associated with PARALLELOGRAMS:

- Opposite sides are congruent

$\overline{KL} \cong \overline{MN}$

$\overline{KN} \cong \overline{LN}$

- Opposite angles are congruent

$\angle NKL \cong \angle LMN$

$\angle KNM \cong \angle MLK$

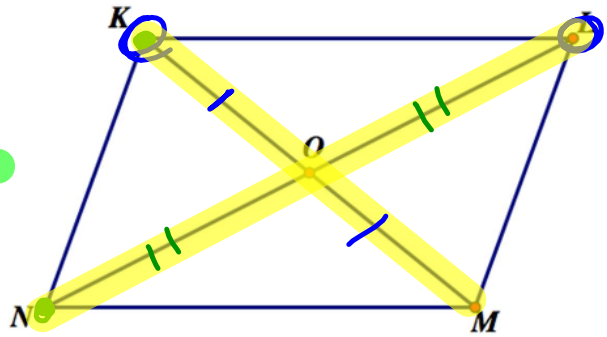
- Consecutive angles are supplementary

$\angle NKL + \angle KLM = 180$

$\angle NKL + \angle KNM = 180$

- Diagonals bisect each other

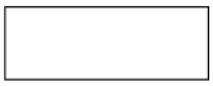
$\overline{KO} \cong \overline{MO}$ $\overline{NO} \cong \overline{LO}$



- Diagonals form two congruent triangles

Parallelograms can be broken down into three more specific types of quadrilaterals with the same properties as parallelograms. The three specific types also have some of their own properties.

Rectangles

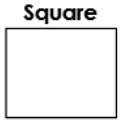


- **All properties of parallelograms**
- Diagonals are congruent
- Four right angles

Rhombus

- **All properties of parallelograms**
- Diagonals are perpendicular
- Diagonals bisect each other
- Four sides are congruent

Square



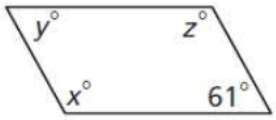
- **All properties of parallelograms**
- Four right angles
- Four congruent sides
- Diagonals are congruent, perpendicular, and bisect each other

1

Geometry Unit 2: Triangle Congruence and Quadrilaterals Notes

Applying Properties of Quadrilaterals

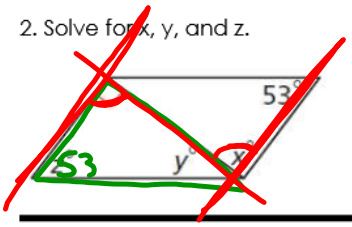
1. Solve for x, y, and z.



Relationship: Opposite <s are ≅
Consecutive <s are Suppl

y = 61°
y + x = 180
61 + x = 180
x = 119°
z = 119°

2. Solve for x, y, and z.



z = 53°

Relationship: opposite <s are ≅

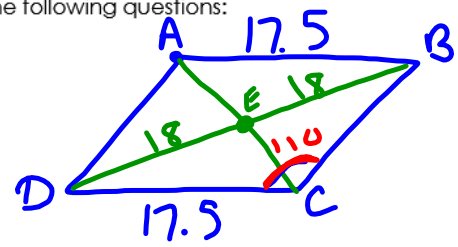
90 + 53 + y = 180
143 + y = 180
y = 37°

x = 90°

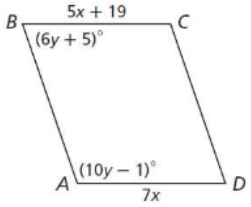
3. In parallelogram ABCD, AB = 17.5, DE = 18, and $m\angle BCD = 110°$. Point E represents the intersection of the diagonals. Draw a picture of parallelogram ABCD and answer the following questions:

a. BD = 36
b. CD = 17.5
c. BE = 18
d. $m\angle ABC$ = 70
e. $m\angle ADC$ = 70
f. $m\angle DAB$ = 110°

m∠ADC + 110 = 180



4. Find the value of x. Then find the length of BC.

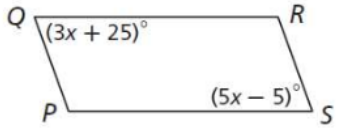


Relationship: opposite side are ≅

5x + 19 = 7x
−5x −5x
19 = 2x

x = 9.5
BC = 5x + 19
BC = 5(9.5) + 19
BC = 66.5

5. Find the value of x. Then find Angle Q.



Relationship: opposite <s are ≅

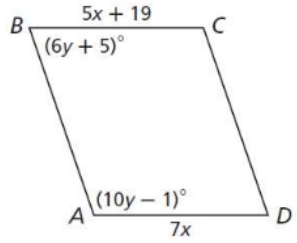
3x + 25 = 5x − 5
−3x −3x
25 = 2x − 5
+5 +5
30 = 2x
x = 15

m∠Q = 3(15) + 25)
m∠Q = 70°

2

Geometry Unit 2: Triangle Congruence and Quadrilaterals Notes

6. Find the value of y. Then find the measure of Angle C and D.



Relationship: Consecutive ∠s are Supplementary

$6y+5+10y-1=180$

$16y+4=180$

$-4 \quad -4$

$16y=176$

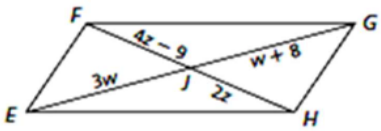
$y=11$

$m\angle B=6(11)+5$

$m\angle B=71$ $m\angle D=71°$

$m\angle C=109°$

7. EFGH is a parallelogram. Find w and z.



Relationship: Diagonals bisect.

$4z-9=2z$

$-4z \quad -4z$

$-9=-2z$

$4.5=z$

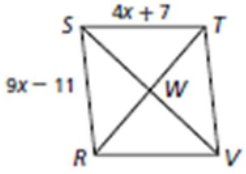
$3w=w+8$

$-w \quad -w$

$2w=8$

$w=4$

8. RSTV is a rhombus. Find the length of TV.



Relationship: Sides are ≅

$ST=TV$

$ST=4(3.6)+7$

$ST=21.4$

$TV=21.4$

$4x+7=9x-11$

$-4x \quad -4x$

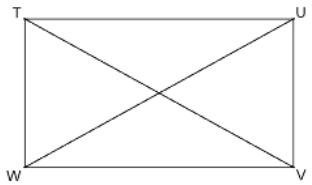
$7=5x-11$

$+11 \quad +11$

$18=5x$

$x=3.6$

9. In rectangle *TUVW* below, it is know that $TV=19-2x$ and $WU=10+x$. Find the value of x.



Relationship: Diagonals are ≅

$19-2x=10+x$

$+2x \quad +2x$

$19=10+3x$

$-10 \quad -10$

$9=3x$

$\frac{9}{3}=\frac{3x}{3}$

$3=x$

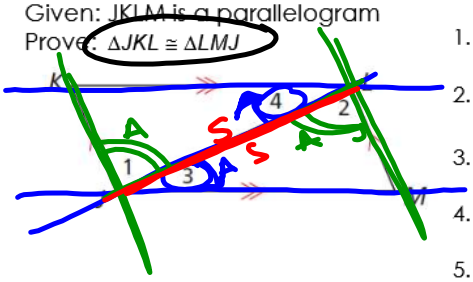
3

Geometry Unit 2: Triangle Congruence and Quadrilaterals Notes

Proving and Justifying with Parallelograms

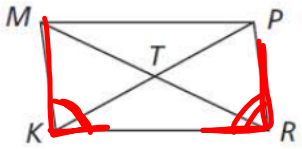
Yesterday, you explored 4 out of the 5 theorems associated with parallelograms. You learned that opposite sides are congruent, opposite angles are congruent, consecutive angles are supplementary, and diagonals bisect each other. It was mentioned that, in a parallelogram, diagonals form two congruent triangles, but you never really explored it. In the problem below, you are going to prove that a parallelogram forms two congruent triangles.

Given: JKLM is a parallelogram
Prove: $\Delta JKL \cong \Delta LMJ$



Statements	Reasons
1. JKLM is a parallelogram	1. Given
2. $\overline{KJ} \parallel \overline{LM}, \overline{KL} \parallel \overline{JM}$	2. Def. of parallelogram
3. $\angle 4 \cong \angle 3$	3. Alternate Interior $\angle$'s are $\cong$
4. $\angle 1 \cong \angle 2$	4. Alt. interior $\angle$s are $\cong$
5. $\overline{JL} \cong \overline{JL}$	5. Reflexive property
6. $\Delta JKL \cong \Delta LMJ$	6. ASA

Using the picture at the right, answer the following questions about parallelogram MPRK. Justify your answer (using properties of parallelograms) for each question.



a. $\angle MPR \cong$ $\angle MKR$ Why? Opposite $\angle$s are $\cong$

b. $\angle PRK \cong$ $\angle PMK$ Why? Opposite $\angle$s are $\cong$

c. $\overline{MT} \cong$ $\overline{TR}$ Why? diagonals bisect Each other

d. $\overline{PR} \cong$ $\overline{MK}$ Why? Opposite Sides are $\cong$

e. $\overline{MP} \parallel$ $\overline{KR}$ Why? Opposite Sides are $\parallel$

f. $\overline{MK} \parallel$ $\overline{PR}$ Why? opposite sides are $\parallel$

g. $\angle MPK \cong$ $\angle PKR$ Why? Alternate interior $\angle$s are $\cong$

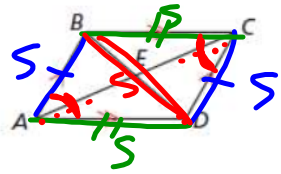
h. $\angle MTK \cong$ $\angle PTR$ Why? Vertical $\angle$s are $\cong$

i. $m\angle MKR + m\angle PRK =$ 180 Why? Consecutive $\angle$s are Supplementary

Geometry Unit 2: Triangle Congruence and Quadrilaterals Notes

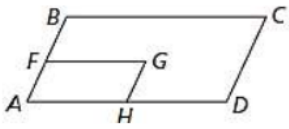
Proofs with Parallelograms

a. **Given:** ABCD is a parallelogram
Prove: $\angle BAD \cong \angle DCB$



Statements	Reasons
1. ABCD is a parallelogram	1. Given
2. $\overline{AB} \cong \overline{CD}$	2. opposites sides of parallelogram are $\cong$
3. $\overline{DA} \cong \overline{BC}$	3. opposite sides of parallelogram are $\cong$
4. $\overline{BD} \cong \overline{BD}$	4. Reflexive property
5. $\triangle BAD \cong \triangle DCB$	5. SSS
6. $\angle BAD \cong \angle DCB$	6. Corresponding parts of $\cong \triangle$ are $\cong$

b. **Given:** ABCD and AFGH are parallelograms
Prove: $\angle C \cong \angle G$

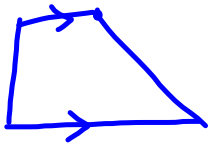


Statements	Reasons
1. ABCD is a parallelogram	1. _____
2. _____	2. Given
3. $\angle C \cong \angle A$	3. _____
4. $\angle A \cong \angle G$	4. _____
5. _____	5. _____

c. **Given:** EFGH is a rectangle, J is the midpoint of $\overline{EH}$.
Prove: ΔFJG is isosceles.

Statements	Reasons
1. EFGH is a rectangle.	1. _____
2. $\angle E$ & $\angle H$ are right angles.	2. _____
3. _____	3. _____
4. J is the midpoint of $\overline{EH}$.	4. _____
5. _____	5. _____
6. EFGH is also a parallelogram	6. _____
7. $\overline{FE} \cong \overline{GH}$	7. _____
8. $\Delta FJE \cong \Delta GJH$	8. _____
9. _____	9. _____
10. _____	10. _____

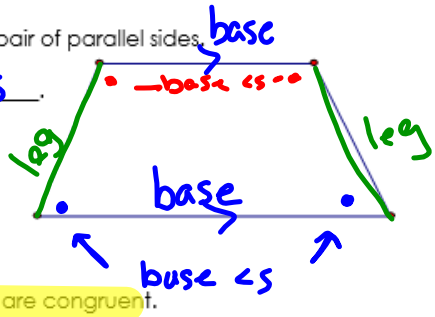
6



Properties of Trapezoids and Kites

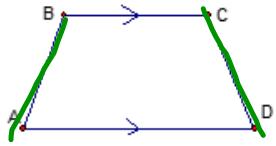
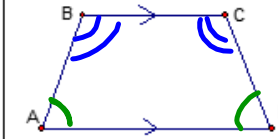
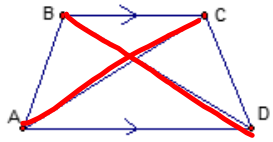
A Trapezoid is a quadrilateral with ***exactly*** 1 pair of parallel sides.

1. The **parallel** sides are called the bases.
2. The **non-parallel** sides are called the legs.
3. The trapezoid has 2 pairs of base angles. Each pair share a side that is a base.



Isosceles Trapezoid- A trapezoid where the legs are congruent.

Properties of isosceles trapezoids

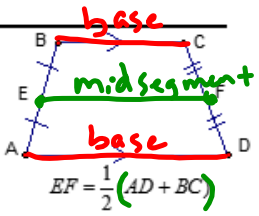
Legs are congruent	Base angles are congruent	Diagonals are congruent
		
$\overline{BA} \cong \overline{CD}$	$\angle BAD \cong \angle CDA$	$\overline{BD} \cong \overline{AC}$

$\angle ABC \cong \angle DCB$

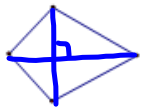
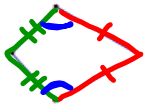
Midsegment Theorem for Trapezoids

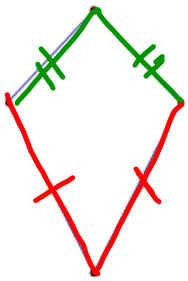
The Midsegment of a Trapezoid is:

1. Parallel to Each Base
2. Has a length equal to $\frac{1}{2}$ the sum of the length of its bases (its length is the average of the bases).



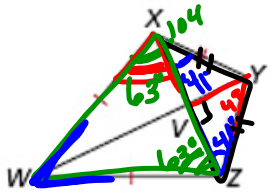
A Kite is a quadrilateral that has two pairs of congruent sides
BUT opposite sides are ***not*** congruent

The diagonals are perpendicular.	
Exactly one pair of opposite angles are congruent.	



In kite WXYZ, m∠WXY = 104°, and m∠VYZ = 49°. Find each measure.

1. m∠VZY = 41°
2. m∠VXW = 63°
3. m∠XWZ = 54°



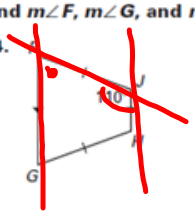
90 + 49 + m∠VZY = 180
139 + m∠VZY = 180
m∠VZY = 41°

104
−41
63°

63 + 63 + m∠XWZ = 180
126 + m∠XWZ = 180
m∠XWZ = 54°

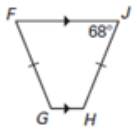
Find m∠F, m∠G, and m∠H.

4.



m∠H = 110°
m∠F + 110 = 180
m∠F = 70°
m∠G = 70°

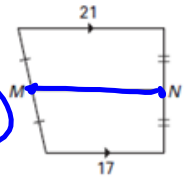
5.



m∠F = 68°
m∠G = 112°
m∠H = 112°

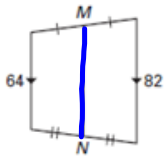
Find the length of the midsegment of the trapezoid.

6.



$MN = \frac{1}{2}(21 + 17)$
$MN = \frac{1}{2}(38)$
MN = 19

7.



$MN = \frac{1}{2}(64 + 82)$
$MN = \frac{1}{2}(146)$
MN = 73

Write the rule for the given transformations.

1. A reflection over the line y = -x.

(+,+) (-,-)

$(-y, -x)$

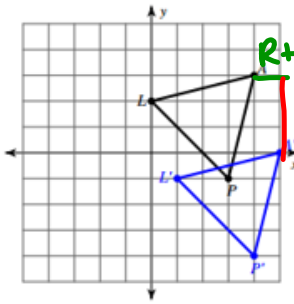
2. A 90° counterclockwise rotation about the origin

$(-y, x)$

(-,+) (+,+)

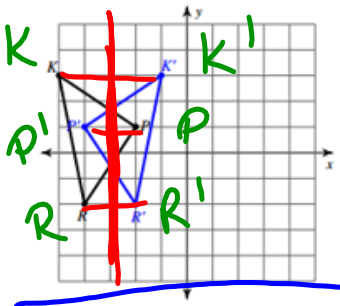
Write the rule to describe each transformation.

3.



$(x+1, y-3)$

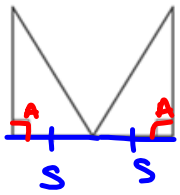
4.



Reflect over $x = -3$

Decide whether it is possible to prove the triangles are congruent. If it is possible, state the theorem or the postulate you would use.

5.

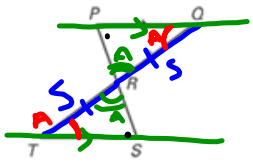


Not ≅

6.

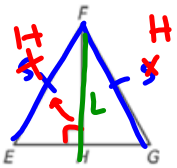
SAS

7.



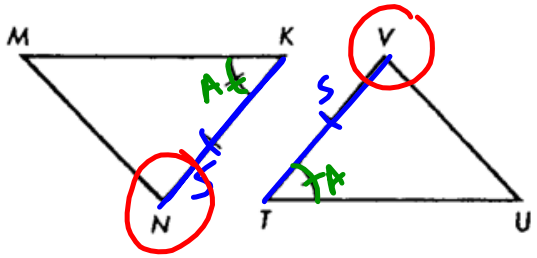
ASA
AAS

8.



HL

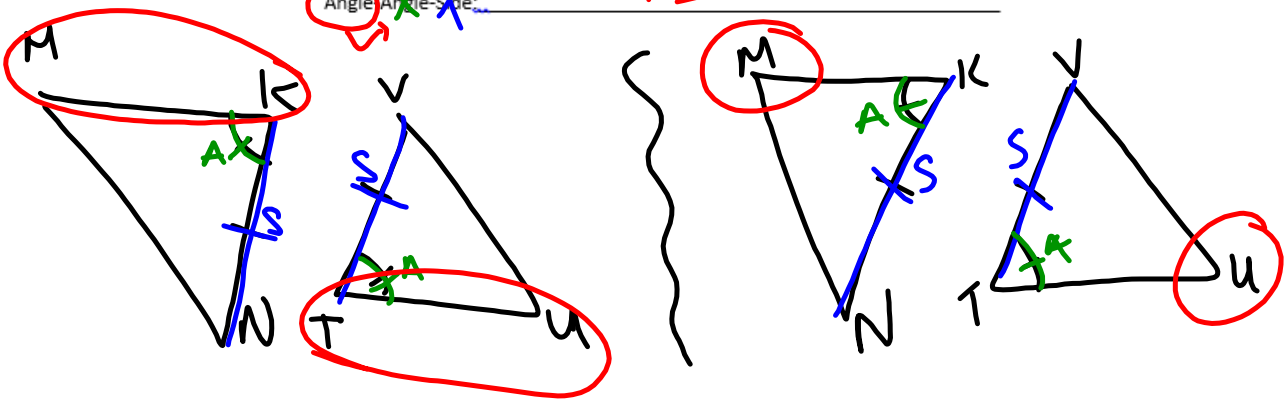
9. Indicate the additional information needed to enable us to apply the specified congruence postulate.



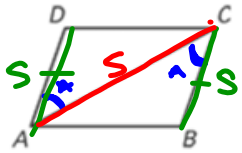
Angle-Side-Angle: $\angle N \cong \angle V$

Side-Angle-Side: $\overline{MK} \cong \overline{TU}$

Angle-Angle-Side: $\angle M \cong \angle U$



10. Complete the following proof.



Given: $\angle CAD \cong \angle ACB$, $\overline{AD} \cong \overline{CB}$
Prove: $\Delta ABC \cong \Delta CDA$

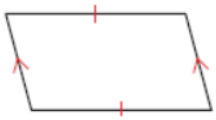
Statements	Reasons
1. $\angle CAD \cong \angle ACB$	1. Given
2. $\overline{AD} \cong \overline{CB}$	2. Given
3. $\overline{AC} \cong \overline{AC}$	3. Reflexive property
4. $\Delta ABC \cong \Delta CDA$	4. SAS

Geometry Unit 2: Triangle Congruence and Quadrilaterals Notes

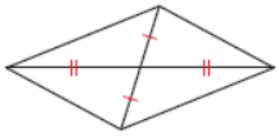
Justifying with Properties of Parallelograms

Determine if each quadrilateral must be a parallelogram. Explain why or why not.

a.



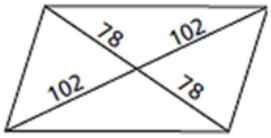
b.



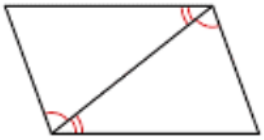
c.

d.

e.



f.



g. If the diagonals are perpendicular, which type of quadrilateral could it be?

h. If all four sides are the same length, which type of quadrilateral could it be?

i. If the diagonals are congruent, which type of quadrilateral could it be?

j. A parallelogram has one right angle. What is a more specific name for the parallelogram? Justify your answer using properties of parallelograms and specific quadrilaterals.